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Higher Order Gabitov–Turitsyn Equation for Dispersion-Managed Solitons in Birefringent Fibers

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Abstract The study of optical solitons in birefringent fibers is governed, to a higher order accuracy, by the higher order Gabitov–Turitsyn equations. These equations are derived by exploiting the multiple scale technique to the vector dispersion-managed nonlinear Schrodinger's equations. The averaged equation, with the higher order terms, drastically improves the description of the soliton characterestics in such fibers.

1 Introduction

The propagation of solitons through optical fibers has been a major area of research given its potential applicability in all optical communication systems [1–20]. The field of telecommunications has undergone a substantial evolution in the last couple of decades due to the impressive progress in the development of optical fibers, optical amplifiers as well as transmitters and receivers. In a modern optical communication system, the transmission link is composed of optical fibers and amplifiers that replace the electrical regenerators. But the amplifiers introduce some noise and signal distortion that limit the system capacity. Presently the optical systems that show the best characteristics in terms of simplicity, cost and robustness against the degrading effects of a link are those based on intensity modulation with direct detection (IM-DD). Conventional IM-DD systems are based on non-return-to-zero (NRZ) format, but for transmission at higher data rate the return-to-zero (RZ) format is preferred. When the data rate is quite high, soliton transmission can be used. It allows the exploitation of the fiber capacity much more, but the NRZ signals offer very high potential especially in terms of simplicity [9].

There are limitations, however, on the performance of optical system due to several effects that are present in optical fibers and amplifiers. Signal propagation through optical fibers can be affected by group velocity dispersion (GVD), polarization mode dispersion (PMD) and the nonlinear effects. The chromatic dispersion that is essentially the GVD

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when waveguide dispersion is negligible, is a linear effect that introduces pulse broadening generates intersymbol intereference. The PMD arises due the fact that optical fibers for telecommunications have two polarization modes, in spite of the fact that they are called monomode fibers. These modes have two different group velocities that induce pulse broadening depending on the input signal state of polarization. The transmission impairment due to PMD looks similar to that of the GVD. However, PMD is a random process as compared to the GVD that is a deterministic process. So PMD cannot be controlled at the receiver. Newly installed optical fibers have quite low values of PMD that is about $0.1\ \mathrm{ps}/\sqrt{\mathrm{km}}$.

The main nonlinear effects that arises in monomode fibers are the Brillouin scattering, Raman scattering and the Kerr effect. Brillouin is a backward scattering that arises from acoustic waves and can generate forward noise at the receiver. Raman scattering is a forward scattering from silica molecules. The Raman gain response is characterized by low gain and wide bandwidth namely about 5 THz. The Raman threshold in conventional fibers is of the order of 500 mW for copolarized pump and Stokes' wave (that is about 1 W for random polarization), thus making Raman effect negligible for a single channel signal. However, it becomes important for multichannel wavelength-division-multiplexed (WDM) signal due to an extremely wide band of wide gain curve.

The Kerr effect of nonlinearity is due to the dependence of the fiber refractive index on the field intensity. This effect mainly manifests as a new frequency when an optical signal propagates through a fiber. In a single channel the Kerr effect induces a spectral broadening and the phase of the signal is modulated according to its power profile. This effect is called self-phase modulation (SPM). The SPM-induced chirp combines with the linear chirp generated by the chromatic dispersion. If the fiber dispersion coefficient is positive namely in the normal dispersion regime, linear and nonlinear chirps have the same sign while in the anomalous dispersion regime they are of opposite signs. In the former case, pulse broadening is enhanced by SPM while in the later case it is reduced. In the anomalous dispersion case the Kerr nonlinearity induces a chirp that can compensate the degradation induced by GVD. Such a compensation is total if soliton signals are used.

If multichannel WDM signals are considered, the Kerr effect can be more degrading since it induces nonlinear cross-talk among the channels that is known as the cross-phase modulation (XPM). In addition WDM generates new frequencies called the Four-Wave mixing (FWM). The other issue in the WDM system is the collision-induced timing jitter that is introduced due to the collision of solitons in different channels. The XPM causes further nonlinear chirp that interacts with the fiber GVD as in the case of SPM. The FWM is a parametric interaction among waves satisfying a particular relationship called phase-matching that lead to power transfer among different channels.

To limit the FWM effect in a WDM it is preferable to operate with a local high GVD that is periodically compensated by devices having an opposite sign of GVD. One such device is a simple optical fiber with opportune GVD and the method is commonly known as the dispersion-management. With this approach the accumulated GVD can be very low and at the same time FWM effect is strongly limited. Through dispersion-management it is possible to achieve highest capacity for both RZ as well as NRZ signals. In that case the overall link dispersion has to be kept very close to zero, while a small amount of chromatic anomalous dispersion is useful for the efficient propagation of a soliton signal. It has been demonstrated that with soliton signals, the dispersion-management is very useful since it reduces collision induced timing jitter [3] and also the pulse interactions. It thus permits the achievement of higher capacities as compared to the link having constant chromatic dispersion.

2 Governing Equations

The relevant equation is the nonlinear Schrödinger's equation (NLSE) with damping and periodic amplification [1, 7] that is given in the dimensionless form as

$$iu_z + \frac{D(z)}{2}u_{tt} + |u|^2 u = -i\Gamma u + i[e^{\Gamma z_a} - 1]\sum_{n=1}^{N}\delta(z - nz_a)u. \tag{1}$$

Here, Γ is the normalized loss coefficient, z_a is the normalized characteristic amplifier spacing and z and t represent the normalized propagation distance and the normalized time, respectively, expressed in the usual nondimensional units.

Also, $D(z)$ is used to model strong dispersion-management. The fiber dispersion $D(z)$ into two components namely a path-averaged constant value δ_a and a term representing the large rapid variation due to large local values of the dispersion [11, 12]. Thus,

$$D(z) = \delta_a + \frac{1}{z_a}\Delta(\zeta) \tag{2}$$

where $\zeta = z/z_a$. The function $\Delta(\zeta)$ is taken to have average zero (namely $\langle\Delta\rangle = 0$), so that the path-averaged dispersion $\langle D\rangle = \delta_a$. The proportionality factor in front of $\Delta(\zeta)$ is chosen so that both δ_a and $\Delta(\zeta)$ are quantities of order one. In practical situations, dispersion-management is often performed by concatenating together two or more sections of given length of a fiber with different values of fiber dispersion. In the special case of a two-step map it is convenient to write the dispersion map as a periodic extension of [12]

$$\Delta(\zeta) = \begin{cases} \Delta_1\colon & 0 \le |\zeta| < \frac{\theta}{2}, \\ \Delta_2\colon & \frac{\theta}{2} \le |\zeta| < \frac{1}{2} \end{cases} \tag{3}$$

where Δ_1 and Δ_2 are given by

$$\Delta_1 = \frac{2s}{\theta} \tag{4}$$

$$\Delta_2 = -\frac{2s}{1-\theta} \tag{5}$$

with the map strength s defined as

$$s = \frac{\theta\Delta_1 - (1-\theta)\Delta_2}{4}. \tag{6}$$

Conversely,

$$s = \frac{\Delta_1\Delta_2}{4(\Delta_2 - \Delta_1)} \tag{7}$$

and

$$\theta = \frac{\Delta_2}{\Delta_2 - \Delta_1}. \tag{8}$$

Now, taking into account the loss and amplification cycles by looking for a solution of (1) of the form $u(z,t) = A(z)q(z,t)$ for real A and letting A satisfy

$$A_z + \Gamma A - [e^{\Gamma z_a} - 1]\sum_{n=1}^{N}\delta(z - nz_a)A = 0 \tag{9}$$

it can be shown that (1) transforms to

$$iq_z + \frac{D(z)}{2}q_{tt} + g(z)|q|^2 q = 0 \tag{10}$$

where

$$g(z) = A^2(z) = a_0^2 e^{-2\Gamma(z-nz_a)} \tag{11}$$

for $z \in [nz_a, (n+1)z_a)$ and $n > 0$ and also

$$a_0 = \left[\frac{2\Gamma z_a}{1 - e^{-2\Gamma z_a}}\right]^{\frac{1}{2}} \tag{12}$$

so that $\langle g(z)\rangle = 1$ over each amplification period [12]. Equation (10) governs the propagation of a dispersion-managed soliton through a polarization preserved optical fiber with damping and periodic amplification [14, 18–20].

3 Birefringent Fibers

A single mode fiber supports two degenerate modes that are polarized in two orthogonal directions. Under ideal conditions of perfect cylindrical geometry and isotropic material, a mode excited with its polarization in one direction would not couple with the mode in the orthogonal direction. However, small deviations from the cylindrical geometry or small fluctuations in material anisotropy result in a mixing of the two polarization states and the mode degeneracy is broken. Thus, the mode propagation constant becomes slightly different for the modes polarized in orthogonal directions. This property is referred to as modal birefringence [16]. Birefringence can also be introduced artificially in optical fibers.

The propagation of solitons in birefringent nonlinear fibers has attracted much attention in recent years. It has potential applications in optical communications and optical logic devices. The equations that describe the pulse propagation through these fibers was originally derived by Menyuk [7]. They can be solved approximately in certain special cases only. The localized pulse evolution in a birefringent fiber has been studied analytically, numerically and experimentally [16] on the basis of a simplified chirp-free model without oscillating terms under the assumptions that the two polarizations exhibit different group velocities. The equations that describe the pulse propagation in birefringent fibers are of the following dimensionless form:

$$i(u_z + \delta u_t) + \beta u + \frac{D(z)}{2}u_{tt} + g(z)(|u|^2 + \alpha|v|^2)u + \gamma v^2 u^* = 0, \tag{13}$$

$$i(v_z - \delta v_t) + \beta v + \frac{D(z)}{2}v_{tt} + g(z)(|v|^2 + \alpha|u|^2)v + \gamma u^2 v^* = 0. \tag{14}$$

Equations (13) and (14) are known as the Dispersion Managed Vector Nonlinear Schrodinger's Equation (DM-VNLSE). Here, u and v are slowly varying envelopes of the two linearly polarized components of the field along the x and y axis. Also, δ is the group velocity mismatch between the two polarization components and is called the birefringence parameter, β corresponds to the difference between the propagation constants, α is the cross-phase modulation (XPM) coefficient and γ is the coefficient of the coherent energy coupling (four-wave mixing) term. These equations are, in general, not integrable. However, they can be solved analytically for certain specific cases only [10, 16].

In this paper, the terms with δ will be neglected as $\delta \leq 10^{-3}$ [13]. Also, neglecting β and the four wave mixing terms given by the coefficient of γ, gives

$$iu_z + \frac{D(z)}{2}u_{tt} + g(z)(|u|^2 + \alpha|v|^2)u = 0, \tag{15}$$

$$iv_z + \frac{D(z)}{2}v_{tt} + g(z)(|v|^2 + \alpha|u|^2)v = 0. \tag{16}$$

Equations (15) and (16) are now going to be studied using the method of multiple-scale perturbation since there is no inverse scattering solution to them.

3.1 Integrals of Motion

The DM-VNLSE has only a couple of conserved quantities namely the energy (E) and the linear momentum (M) of the pulses that are respectively given by

$$E = \int_{-\infty}^{\infty} (|u|^2 + |v|^2)dt \tag{17}$$

$$M = \frac{i}{2}D(z)\int_{-\infty}^{\infty} (u^* u_t - uu_t^* + v^* v_t - vv_t^*)dt \tag{18}$$

The Hamiltonian (H) which is given by

$$\begin{aligned} H = \frac{1}{2}\int_{-\infty}^{\infty}\bigg[&\frac{D(z)}{2}(|u_t|^2 + |v_t|^2) - \beta(|u|^2 - |v|^2) - \frac{g(z)}{2}(|u|^4 + |v|^4) \\ &- i\frac{\delta}{2}(u^* u_t - uu_t^* + v^* v_t - vv_t^*) - \alpha|u|^2|v|^2 \\ &- \frac{1}{2}(1-\alpha)(u^2 v^{*2} + v^2 u^{*2})\bigg]dt \end{aligned} \tag{19}$$

is however not a constant of motion, in general unless $D(z)$ and $g(z)$ are constants. For the reduced set of equation given by (15) and (16), the Hamiltonian is

$$\begin{aligned} H = \int_{-\infty}^{\infty}\bigg[&\frac{D(z)}{2}(|u_t|^2 + |v_t|^2) \\ &- \frac{g(z)}{2}(|u|^4 + |v|^4) - \alpha|u|^2|v|^2 - \frac{1}{2}(1-\alpha)(u^2 v^{*2} + v^2 u^{*2})\bigg]dt. \end{aligned} \tag{20}$$

The existence of a Hamiltonian implies that (15) and (16) can be written as

$$i\frac{\partial u}{\partial z}=\frac{\delta H}{\delta u^*} \tag{21}$$

and

$$i\frac{\partial v}{\partial z}=\frac{\delta H}{\delta v^*}. \tag{22}$$

This defines a Hamiltonian dynamical system on an infinite-dimensional phase space of two complex functions u and v that decrease to zero at infinity and can be analyzed using the theory of Hamiltonian system.

3.2 Asymptotic Analysis

Equations (15) and (16) contains both large and rapidly varying terms. To obtain the asymptotic behavior the fast and slow z scales are introduced as

$$\zeta=\frac{z}{z_a} \tag{23}$$

and

$$Z=z. \tag{24}$$

The fields u and v are expanded in powers of z_a as

$$u(\zeta,Z,t)=u^{(0)}(\zeta,Z,t)+z_a u^{(1)}(\zeta,Z,t)+z_a^2 u^{(2)}(\zeta,Z,t)+\cdots, \tag{25}$$

$$v(\zeta,Z,t)=v^{(0)}(\zeta,Z,t)+z_a v^{(1)}(\zeta,Z,t)+z_a^2 v^{(2)}(\zeta,Z,t)+\cdots. \tag{26}$$

Equating coefficients of like powers of z_a gives

$$O\left(\frac{1}{z_a}\right):\quad i\frac{\partial u^{(0)}}{\partial \zeta}+\frac{\Delta(\zeta)}{2}\frac{\partial^2 u^{(0)}}{\partial t^2}=0 \tag{27}$$

$$O\left(\frac{1}{z_a}\right):\quad i\frac{\partial v^{(0)}}{\partial \zeta}+\frac{\Delta(\zeta)}{2}\frac{\partial^2 v^{(0)}}{\partial t^2}=0 \tag{28}$$

$$O(1):\quad i\frac{\partial u^{(1)}}{\partial \zeta}+\frac{\Delta(\zeta)}{2}\frac{\partial^2 u^{(1)}}{\partial t^2} + \left\{i\frac{\partial u^{(0)}}{\partial Z}+\frac{\delta_a}{2}\frac{\partial^2 u^{(0)}}{\partial t^2}+g(z)(|u^{(0)}|^2+\alpha|v^{(0)}|^2)u^{(0)}\right\}=0, \tag{29}$$

$$O(1):\quad i\frac{\partial v^{(1)}}{\partial \zeta}+\frac{\Delta(\zeta)}{2}\frac{\partial^2 v^{(1)}}{\partial t^2} + \left\{i\frac{\partial v^{(0)}}{\partial Z}+\frac{\delta_a}{2}\frac{\partial^2 v^{(0)}}{\partial t^2}+g(z)(|v^{(0)}|^2+\alpha|u^{(0)}|^2)v^{(0)}\right\}=0, \tag{30}$$

$$O(z_a): \quad i\frac{\partial u^{(2)}}{\partial \zeta} + \frac{\Delta(\zeta)}{2}\frac{\partial^2 u^{(2)}}{\partial t^2} + \left\{ i\frac{\partial u^{(1)}}{\partial Z} + \frac{\delta_a}{2}\frac{\partial^2 u^{(1)}}{\partial t^2} + g(z)[2|u^{(0)}|^2 u^{(1)} + (u^{(0)})^2 (u^{(1)})^* + \alpha\{2|v^{(0)}|^2 v^{(1)} + (v^{(0)})^2 (v^{(1)})^*\}] \right\}, \tag{31}$$

$$O(z_a): \quad i\frac{\partial v^{(2)}}{\partial \zeta} + \frac{\Delta(\zeta)}{2}\frac{\partial^2 v^{(2)}}{\partial t^2} + \left\{ i\frac{\partial v^{(1)}}{\partial Z} + \frac{\delta_a}{2}\frac{\partial^2 v^{(1)}}{\partial t^2} + g(z)[2|v^{(0)}|^2 v^{(1)} + (v^{(0)})^2 (v^{(1)})^* + \alpha\{2|u^{(0)}|^2 u^{(1)} + (u^{(0)})^2 (u^{(1)})^*\}] \right\}. \tag{32}$$

Now the Fourier transform and its inverse are respectively defined as

$$\hat{f}(\omega) = \mathbf{F}[f] \equiv \int_{-\infty}^{\infty} f(t) e^{i\omega t} dt, \tag{33}$$

$$f(t) = \mathbf{F}^{-1}[\hat{f}] \equiv \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{f}(\omega) e^{-i\omega t} d\omega. \tag{34}$$

At $O(1/z_a)$ equations (27) and (28), respectively, in the Fourier domain are given by

$$i\frac{\partial \hat{u}^{(0)}}{\partial \zeta} - \frac{\omega^2}{2}\Delta(\zeta)\hat{u}^{(0)} = 0 \tag{35}$$

and

$$i\frac{\partial \hat{v}^{(0)}}{\partial \zeta} - \frac{\omega^2}{2}\Delta(\zeta)\hat{v}^{(0)} = 0 \tag{36}$$

whose respective solutions are

$$\hat{u}^{(0)}(\zeta, Z, \omega) = \hat{U}_0(Z, \omega) e^{-\frac{i\omega^2}{2}C(\zeta)} \tag{37}$$

and

$$\hat{v}^{(0)}(\zeta, Z, \omega) = \hat{V}_0(Z, \omega) e^{-\frac{i\omega^2}{2}C(\zeta)} \tag{38}$$

where

$$\hat{U}_0(Z, \omega) = \hat{u}^{(0)}(0, Z, \omega), \tag{39}$$

$$\hat{V}_0(Z, \omega) = \hat{v}^{(0)}(0, Z, \omega) \tag{40}$$

and

$$C(\zeta) = \int_0^{\zeta} \Delta(\zeta') d\zeta'. \tag{41}$$

At $O(1)$, equations (29) and (30) are solved in the Fourier domain by substituting the respective solutions given by (37) and (38) for $u^{(0)}$ and $v^{(0)}$ in (29) and (30). This gives

$$i\frac{\partial \hat{u}^{(1)}}{\partial \zeta} - \frac{\omega^2}{2}\Delta(\zeta)\hat{u}^{(1)} = -e^{-\frac{i\omega^2}{2}C(\zeta)}\left(\frac{\partial \hat{U}_0}{\partial Z} - \frac{\omega^2}{2}\delta_a \hat{U}_0\right) - g(\zeta)\int_{-\infty}^{\infty}(|u^{(0)}|^2 + \alpha |v^{(0)}|^2)u^{(0)}e^{i\omega t}dt \quad (42)$$

and

$$i\frac{\partial \hat{v}^{(1)}}{\partial \zeta} - \frac{\omega^2}{2}\Delta(\zeta)\hat{v}^{(1)} = -e^{-\frac{i\omega^2}{2}C(\zeta)}\left(\frac{\partial \hat{V}_0}{\partial Z} - \frac{\omega^2}{2}\delta_a \hat{V}_0\right) - g(\zeta)\int_{-\infty}^{\infty}(|v^{(0)}|^2 + \alpha |u^{(0)}|^2)v^{(0)}e^{i\omega t}dt. \quad (43)$$

Equations (42) and (43) are inhomogeneous equations for $\hat{u}^{(1)}$ and $\hat{v}^{(1)}$ respectively with the homogeneous parts having the same structures as in (35) and (36) respectively. For the non-secularity conditions of $\hat{u}^{(1)}$ and $\hat{v}^{(1)}$, it is necessary that the forcing terms be orthogonal to the adjoint solutions of (35) and (36) respectively, a condition that is commonly known as Fredholm's Alternative (FA). This gives the conditions for $\hat{U}_0(Z,\omega)$ and $\hat{V}_0(Z,\omega)$ respectively as

$$\frac{\partial \hat{U}_0}{\partial Z} - \frac{\omega^2}{2}\delta_a \hat{U}_0 + \int_0^1\int_{-\infty}^{\infty} e^{\frac{i\omega^2}{2}C(\zeta)}g(\zeta)(|u^{(0)}|^2 + \alpha|v^{(0)}|^2)u^{(0)}e^{i\omega t}dt\,d\zeta = 0 \quad (44)$$

and

$$\frac{\partial \hat{V}_0}{\partial Z} - \frac{\omega^2}{2}\delta_a \hat{V}_0 + \int_0^1\int_{-\infty}^{\infty} e^{\frac{i\omega^2}{2}C(\zeta)}g(\zeta)(|v^{(0)}|^2 + \alpha|u^{(0)}|^2)v^{(0)}e^{i\omega t}dt\,d\zeta = 0 \quad (45)$$

Equations (44) and (45) can be respectively simplified to

$$\frac{\partial \hat{U}_0}{\partial Z} - \frac{\omega^2}{2}\delta_a \hat{U}_0 + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} r_0(\omega_1\omega_2)\hat{U}_0(Z,\omega_1+\omega_2)[\hat{U}_0(Z,\omega+\omega_1)\hat{U}_0(Z,\omega+\omega_1+\omega_2) + \alpha\hat{V}_0(Z,\omega+\omega_1)\hat{V}_0(Z,\omega+\omega_1+\omega_2)]d\omega_1 d\omega_2 = 0 \quad (46)$$

and

$$\frac{\partial \hat{V}_0}{\partial Z} - \frac{\omega^2}{2}\delta_a \hat{V}_0 + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} r_0(\omega_1\omega_2)\hat{V}_0(Z,\omega_1+\omega_2)[\hat{V}_0(Z,\omega+\omega_1)\hat{V}_0(Z,\omega+\omega_1+\omega_2) + \alpha\hat{U}_0(Z,\omega+\omega_1)\hat{U}_0(Z,\omega+\omega_1+\omega_2)]d\omega_1 d\omega_2 = 0 \quad (47)$$

where, the kernel $r_0(x)$ is given by

$$r_0(x) = \frac{1}{(2\pi)^2}\int_0^1 g(\zeta)e^{ixC(\zeta)}d\zeta. \tag{48}$$

Equations (46) and (47) are commonly known as the Gabitov–Turitsyn equations (GTE) for the propagation of solitons through a birefringent fiber.

Equations (42) and (43) will now be solved to obtain $u^{(1)}(\zeta, Z, t)$ and $v^{(1)}(\zeta, Z, t)$. Substituting $\hat{U}_0$ and $\hat{V}_0$ into the right side of (42) and (43) respectively and using the pairs (37–38) and (44–45) gives

$$\begin{aligned}
&\frac{\partial}{\partial\zeta}[i\hat{u}^{(1)}e^{\frac{i\omega^2}{2}C(\zeta)}] \\
&\quad = \int_0^1\int_{-\infty}^{\infty} g(\zeta)e^{\frac{i\omega^2}{2}C(\zeta)}(|u^{(0)}|^2 + \alpha|v^{(0)}|^2)u^{(0)}e^{i\omega t}dt\,d\zeta \\
&\qquad - g(\zeta)e^{\frac{i\omega^2}{2}C(\zeta)}\int_{-\infty}^{\infty}(|u^{(0)}|^2 + \alpha|v^{(0)}|^2)u^{(0)}e^{i\omega t}dt
\end{aligned} \tag{49}$$

and

$$\begin{aligned}
&\frac{\partial}{\partial\zeta}[i\hat{v}^{(1)}e^{\frac{i\omega^2}{2}C(\zeta)}] \\
&\quad = \int_0^1\int_{-\infty}^{\infty} g(\zeta)e^{\frac{i\omega^2}{2}C(\zeta)}(|v^{(0)}|^2 + \alpha|u^{(0)}|^2)v^{(0)}e^{i\omega t}dt\,d\zeta \\
&\qquad - g(\zeta)e^{\frac{i\omega^2}{2}C(\zeta)}\int_{-\infty}^{\infty}(|v^{(0)}|^2 + \alpha|u^{(0)}|^2)v^{(0)}e^{i\omega t}dt.
\end{aligned} \tag{50}$$

Integration of equations (49) and (50) yields

$$\begin{aligned}
i\hat{u}^{(1)}e^{\frac{i\omega^2}{2}C(\zeta)} = {} & \hat{U}_1(Z,\omega) \\
& + \zeta\int_0^1\int_{-\infty}^{\infty} g(\zeta)e^{\frac{i\omega^2}{2}C(\zeta)}(|u^{(0)}|^2 + \alpha|v^{(0)}|^2)u^{(0)}e^{i\omega t}dt\,d\zeta \\
& - \int_0^{\zeta}\int_{-\infty}^{\infty} g(\zeta')e^{\frac{i\omega^2}{2}C(\zeta')}(|u^{(0)}|^2 + \alpha|v^{(0)}|^2)u^{(0)}e^{i\omega t}dt\,d\zeta'
\end{aligned} \tag{51}$$

and

$$\begin{aligned}
i\hat{v}^{(1)}e^{\frac{i\omega^2}{2}C(\zeta)} = {} & \hat{V}_1(Z,\omega) \\
& + \zeta\int_0^1\int_{-\infty}^{\infty} g(\zeta)e^{\frac{i\omega^2}{2}C(\zeta)}(|v^{(0)}|^2 + \alpha|u^{(0)}|^2)v^{(0)}e^{i\omega t}dt\,d\zeta \\
& - \int_0^{\zeta}\int_{-\infty}^{\infty} g(\zeta')e^{\frac{i\omega^2}{2}C(\zeta')}(|v^{(0)}|^2 + \alpha|u^{(0)}|^2)v^{(0)}e^{i\omega t}dt\,d\zeta'
\end{aligned} \tag{52}$$

where

$$\hat{U}_1(Z,\omega) = i\hat{u}^{(1)}(0, Z, \omega)e^{\frac{i\omega^2}{2}C(0)} \tag{53}$$

and

$$\hat{V}_1(Z,\omega) = i\hat{v}^{(1)}(0,Z,\omega)e^{\frac{i\omega^2}{2}C(0)}. \tag{54}$$

Also, $\hat{U}_1(Z,\omega)$ and $\hat{V}_1(Z,\omega)$ are so chosen that

$$\int_0^1 i\hat{u}^{(1)}e^{\frac{i\omega^2}{2}C(\zeta)}d\zeta = 0 \tag{55}$$

and

$$\int_0^1 i\hat{v}^{(1)}e^{\frac{i\omega^2}{2}C(\zeta)}d\zeta = 0 \tag{56}$$

which are going to be useful relations at subsequent orders. Applying (55) and (56) to (51) and (52) respectively gives

$$\begin{aligned}\hat{U}_1(Z,\omega) = &\int_0^1\int_0^\zeta\int_{-\infty}^{\infty} g(\zeta')e^{\frac{i\omega^2}{2}C(\zeta')}(|u^{(0)}|^2+\alpha|v^{(0)}|^2)u^{(0)}e^{i\omega t}dtd\zeta'd\zeta\\ &-\frac{1}{2}\int_0^1\int_{-\infty}^{\infty} g(\zeta)e^{\frac{i\omega^2}{2}C(\zeta)}(|u^{(0)}|^2+\alpha|v^{(0)}|^2)u^{(0)}e^{i\omega t}dtd\zeta\end{aligned} \tag{57}$$

and

$$\begin{aligned}\hat{V}_1(Z,\omega) = &\int_0^1\int_0^\zeta\int_{-\infty}^{\infty} g(\zeta')e^{\frac{i\omega^2}{2}C(\zeta')}(|v^{(0)}|^2+\alpha|u^{(0)}|^2)v^{(0)}e^{i\omega t}dtd\zeta'd\zeta\\ &-\frac{1}{2}\int_0^1\int_{-\infty}^{\infty} g(\zeta)e^{\frac{i\omega^2}{2}C(\zeta)}(|v^{(0)}|^2+\alpha|u^{(0)}|^2)v^{(0)}e^{i\omega t}dtd\zeta.\end{aligned} \tag{58}$$

Now, (51) and (52), by virtue of (57) and (58) can be respectively written as

$$\begin{aligned}\hat{u}^{(1)}(\zeta,Z,\omega) = ie^{\frac{i\omega^2}{2}C(\zeta)}\Bigg[&\int_0^\zeta\int_{-\infty}^{\infty} g(\zeta')e^{\frac{i\omega^2}{2}C(\zeta')}(|u^{(0)}|^2+\alpha|v^{(0)}|^2)u^{(0)}e^{i\omega t}dtd\zeta'\\ &-\int_0^1\int_0^\zeta\int_{-\infty}^{\infty} g(\zeta')e^{\frac{i\omega^2}{2}C(\zeta')}(|u^{(0)}|^2+\alpha|v^{(0)}|^2)u^{(0)}e^{i\omega t}dtd\zeta'd\zeta\\ &-\left(\zeta-\frac{1}{2}\right)\int_0^1\int_{-\infty}^{\infty} g(\zeta)e^{\frac{i\omega^2}{2}C(\zeta)}(|u^{(0)}|^2+\alpha|v^{(0)}|^2)u^{(0)}e^{i\omega t}dtd\zeta\Bigg]\end{aligned} \tag{59}$$

and

$$\begin{aligned}\hat{v}^{(1)}(\zeta,Z,\omega) = ie^{\frac{i\omega^2}{2}C(\zeta)}\Bigg[&\int_0^\zeta\int_{-\infty}^{\infty} g(\zeta')e^{\frac{i\omega^2}{2}C(\zeta')}(|v^{(0)}|^2+\alpha|u^{(0)}|^2)v^{(0)}e^{i\omega t}dtd\zeta'\\ &-\int_0^1\int_0^\zeta\int_{-\infty}^{\infty} g(\zeta')e^{\frac{i\omega^2}{2}C(\zeta')}(|v^{(0)}|^2+\alpha|u^{(0)}|^2)v^{(0)}e^{i\omega t}dtd\zeta'd\zeta\\ &-\left(\zeta-\frac{1}{2}\right)\int_0^1\int_{-\infty}^{\infty} g(\zeta)e^{\frac{i\omega^2}{2}C(\zeta)}(|v^{(0)}|^2+\alpha|u^{(0)}|^2)v^{(0)}e^{i\omega t}dtd\zeta\Bigg].\end{aligned} \tag{60}$$

The pair (59) and (60) can now be respectively further written as

$$\hat{u}^{(1)}(\zeta, Z, \omega) = ie^{-\frac{i\omega^2}{2}C(\zeta)}\Bigg[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\hat{U}_0^*(\omega+\Omega_1+\Omega_2)\{\hat{U}_0(\omega+\Omega_1)\hat{U}_0(\omega+\Omega_2)$$
$$+\alpha\hat{V}_0(\omega+\Omega_1)\hat{V}_0(\omega+\Omega_2)\}$$
$$\times\left\{\int_0^{\zeta}g(\zeta')e^{i\Omega_1\Omega_2C(\zeta')}d\zeta' - \int_0^1\int_0^{\zeta}g(\zeta')e^{i\Omega_1\Omega_2C(\zeta')}d\zeta' d\zeta\right.$$
$$\left.-\left(\zeta-\frac{1}{2}\right)\int_0^1 g(\zeta)e^{i\Omega_1\Omega_2C(\zeta)}d\zeta\right\}d\Omega_1\Omega_2\Bigg] \tag{61}$$

and

$$\hat{v}^{(1)}(\zeta, Z, \omega) = ie^{-\frac{i\omega^2}{2}C(\zeta)}\Bigg[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\hat{V}_0^*(\omega+\Omega_1+\Omega_2)\{\hat{V}_0(\omega+\Omega_1)\hat{V}_0(\omega+\Omega_2)$$
$$+\alpha\hat{U}_0(\omega+\Omega_1)\hat{U}_0(\omega+\Omega_2)\}$$
$$\times\left\{\int_0^{\zeta}g(\zeta')e^{i\Omega_1\Omega_2C(\zeta')}d\zeta' - \int_0^1\int_0^{\zeta}g(\zeta')e^{i\Omega_1\Omega_2C(\zeta')}d\zeta' d\zeta\right.$$
$$\left.-\left(\zeta-\frac{1}{2}\right)\int_0^1 g(\zeta)e^{i\Omega_1\Omega_2C(\zeta)}d\zeta\right\}d\Omega_1\Omega_2\Bigg]. \tag{62}$$

Thus, at $O(z_a)$,

$$\hat{u}(\zeta, Z, \omega) = \hat{u}^{(0)}(\zeta, Z, \omega) + z_a\hat{u}^{(1)}(\zeta, Z, \omega) \tag{63}$$

and

$$\hat{v}(\zeta, Z, \omega) = \hat{v}^{(0)}(\zeta, Z, \omega) + z_a\hat{v}^{(1)}(\zeta, Z, \omega). \tag{64}$$

Moving on to the next order at $O(z_a^2)$, one can note that the GTE given by (46) and (47) are allowed to have an additional term of $O(z_a)$ such as

$$\frac{\partial\hat{U}_0}{\partial Z} - \frac{\omega^2}{2}\delta_a\hat{U}_0$$
$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}r_0(\omega_1\omega_2)\hat{U}_0(Z,\omega_1+\omega_2)[\hat{U}_0(Z,\omega+\omega_1)\hat{U}_0(Z,\omega+\omega_1+\omega_2)$$
$$+\alpha\hat{V}_0(Z,\omega+\omega_1)\hat{V}_0(Z,\omega+\omega_1+\omega_2)]d\omega_1 d\omega_2 = z_a\hat{n}_1(Z,\omega) + O(z_a^2) \tag{65}$$

and

$$\frac{\partial\hat{V}_0}{\partial Z} - \frac{\omega^2}{2}\delta_a\hat{V}_0$$
$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}r_0(\omega_1\omega_2)\hat{V}_0(Z,\omega_1+\omega_2)[\hat{V}_0(Z,\omega+\omega_1)\hat{V}_0(Z,\omega+\omega_1+\omega_2)$$
$$+\alpha\hat{U}_0(Z,\omega+\omega_1)\hat{U}_0(Z,\omega+\omega_1+\omega_2)]d\omega_1 d\omega_2 = z_a\hat{n}_2(Z,\omega) + O(z_a^2). \tag{66}$$

The higher order corrections $\hat{n}_1$ and $\hat{n}_2$ can be obtained from suitable non-secular conditions at $O(z_a^2)$ in (31) and (32) respectively. Now, equations (31) and (32), in the Fourier domain, respectively are

$$\frac{\partial}{\partial\zeta}[i\hat{u}^{(2)}e^{\frac{i\omega^2}{2}C(\zeta)}]+\hat{n}_1+e^{\frac{i\omega^2}{2}C(\zeta)}\left(i\frac{\partial\hat{u}^{(1)}}{\partial Z}-\frac{\omega^2}{2}\delta_a\hat{u}^{(1)}\right)$$
$$+e^{\frac{i\omega^2}{2}C(\zeta)}g(\zeta)\int_{-\infty}^{\infty}[2|u^{(0)}|^2u^{(1)}+(u^{(0)})^2u^{(1)*}$$
$$+\alpha\{2|v^{(0)}|^2v^{(1)}+(v^{(0)})^2v^{(1)*}\}]e^{i\omega t}dt=0 \tag{67}$$

and

$$\frac{\partial}{\partial\zeta}[i\hat{v}^{(2)}e^{\frac{i\omega^2}{2}C(\zeta)}]+\hat{n}_2+e^{\frac{i\omega^2}{2}C(\zeta)}\left(i\frac{\partial\hat{v}^{(1)}}{\partial Z}-\frac{\omega^2}{2}\delta_a\hat{v}^{(1)}\right)$$
$$+e^{\frac{i\omega^2}{2}C(\zeta)}g(\zeta)\int_{-\infty}^{\infty}[2|v^{(0)}|^2v^{(1)}+(v^{(0)})^2v^{(1)*}$$
$$+\alpha\{2|u^{(0)}|^2u^{(1)}+(u^{(0)})^2u^{(1)*}\}]e^{i\omega t}dt=0. \tag{68}$$

But, again (55) and (56) gives

$$\int_0^1\hat{u}^{(1)}e^{\frac{i\omega^2}{2}C(\zeta)}d\zeta=0 \tag{69}$$

and

$$\int_0^1\hat{v}^{(1)}e^{\frac{i\omega^2}{2}C(\zeta)}d\zeta=0. \tag{70}$$

Applying the non-secularity conditions (69) and (70) to (67) and (68) respectively gives

$$\hat{n}_1=-\int_0^1\int_{-\infty}^{\infty}e^{\frac{i\omega^2}{2}C(\zeta)}g(\zeta)$$
$$\times[2|u^{(0)}|^2u^{(1)}+(u^{(0)})^2u^{(1)*}+\alpha\{2|v^{(0)}|^2v^{(1)}+(v^{(0)})^2v^{(1)*}\}]e^{i\omega t}dt\,d\zeta \tag{71}$$

and

$$\hat{n}_2=-\int_0^1\int_{-\infty}^{\infty}e^{\frac{i\omega^2}{2}C(\zeta)}g(\zeta)$$
$$\times[2|v^{(0)}|^2v^{(1)}+(v^{(0)})^2v^{(1)*}+\alpha\{2|u^{(0)}|^2u^{(1)}+(u^{(0)})^2u^{(1)*}\}]e^{i\omega t}dt\,d\zeta \tag{72}$$

Using the pairs (37–38) and (59–60), equations (71) and (72) can respectively be written as

$$\hat{n}_1=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}r_1(\omega_1\omega_2,\Omega_1\Omega_2)$$
$$\times[\{2\hat{U}_0(\omega+\omega_1)\hat{U}_0^*(\omega+\omega_1+\omega_2)\hat{U}_0(\omega+\omega_2+\Omega_1)$$
$$\times\hat{U}_0(\omega+\omega_2+\Omega_2)\hat{U}_0^*(\omega+\omega_2+\Omega_1+\Omega_2)$$

$$
\begin{aligned}
&- \hat{U}_0(\omega+\omega_1)\hat{U}_0(\omega+\omega_2)\hat{U}_0^*(\omega+\omega_1+\omega_2+\Omega_1)\\
&\times \hat{U}_0^*(\omega+\omega_1+\omega_2-\Omega_2)\hat{U}_0^*(\omega+\omega_1+\omega_2+\Omega_1-\Omega_2)\}\\
&+\alpha\{2\hat{V}_0(\omega+\omega_1)\hat{V}_0^*(\omega+\omega_1+\omega_2)\hat{V}_0(\omega+\omega_2+\Omega_1)\\
&\times \hat{V}_0(\omega+\omega_2+\Omega_2)\hat{V}_0^*(\omega+\omega_2+\Omega_1+\Omega_2)\\
&- \hat{V}_0(\omega+\omega_1)\hat{V}_0(\omega+\omega_2)\hat{V}_0^*(\omega+\omega_1+\omega_2+\Omega_1)\\
&\times \hat{V}_0^*(\omega+\omega_1+\omega_2-\Omega_2)\hat{V}_0^*(\omega+\omega_1+\omega_2+\Omega_1-\Omega_2)\}]d\omega_1 d\omega_2 d\Omega_1 d\Omega_2 \qquad (73)
\end{aligned}
$$

and

$$
\begin{aligned}
\hat{n}_2 = &\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} r_1(\omega_1\omega_2,\Omega_1\Omega_2)\\
&\times [\{2\hat{V}_0(\omega+\omega_1)\hat{V}_0^*(\omega+\omega_1+\omega_2)\hat{V}_0(\omega+\omega_2+\Omega_1)\\
&\times \hat{V}_0(\omega+\omega_2+\Omega_2)\hat{V}_0^*(\omega+\omega_2+\Omega_1+\Omega_2)\\
&- \hat{V}_0(\omega+\omega_1)\hat{V}_0(\omega+\omega_2)\hat{V}_0^*(\omega+\omega_1+\omega_2+\Omega_1)\\
&\times \hat{V}_0^*(\omega+\omega_1+\omega_2-\Omega_2)\hat{V}_0^*(\omega+\omega_1+\omega_2+\Omega_1-\Omega_2)\}\\
&+\alpha\{2\hat{U}_0(\omega+\omega_1)\hat{U}_0^*(\omega+\omega_1+\omega_2)\hat{U}_0(\omega+\omega_2+\Omega_1)\\
&\times \hat{U}_0(\omega+\omega_2+\Omega_2)\hat{U}_0^*(\omega+\omega_2+\Omega_1+\Omega_2)\\
&- \hat{U}_0(\omega+\omega_1)\hat{U}_0(\omega+\omega_2)\hat{U}_0^*(\omega+\omega_1+\omega_2+\Omega_1)\\
&\times \hat{U}_0^*(\omega+\omega_1+\omega_2-\Omega_2)\hat{U}_0^*(\omega+\omega_1+\omega_2+\Omega_1-\Omega_2)\}]d\omega_1 d\omega_2 d\Omega_1 d\Omega_2 \qquad (74)
\end{aligned}
$$

where, the kernel $r_1(x, y)$ is given by

$$
\begin{aligned}
r_1(x,y) = &\frac{1}{(2\pi)^4}\int_0^1\int_0^{\zeta} g(\zeta)g(\zeta')e^{i(xC(\zeta)+yC(\zeta))}d\zeta d\zeta'\\
&-\left[\int_0^1 g(\zeta)e^{ixC(\zeta)}d\zeta\right]\left[\int_0^{\zeta} g(\zeta')e^{ixC(\zeta')}d\zeta'\right]\\
&-\left[\int_0^1\left(\zeta-\frac{1}{2}\right)g(\zeta)e^{ixC(\zeta)}d\zeta\right]\left[\int_0^1 g(\zeta)e^{iyC(\zeta)}d\zeta\right]. \qquad (75)
\end{aligned}
$$

Equations (65) and (66) represent the higher order GTE (HO-GTE) for the propagation of solitons through birefringent optical fibers.

4 Properties of the Kernel

The HO-GTE, for different types of fibers, are the fundamental equations that govern the evolution of optical pulses for a strong dispersion-managed soliton systems corresponding to the frequency and time domain respectively. In these GT equations, all the fast variations and large quantities are removed and so they contain only slowly varying quantities of order one. These equations are not limited to the case $\delta_a > 0$, however, they are also applicable to the case of pulse dynamics with zero or normal values of average dispersion. If the fiber dispersion is constant namely if $\Delta(\zeta) = 0$, then $C(\zeta) = 0$ and so $r_0(x) = 1/(2\pi)^2$ The kernels $r_0(x)$ and $r_1(x, y)$ are now going to be studied in the following two cases

4.1 Lossless Case

For the lossless case, namely when $g(\zeta) = 1$, the kernels $r_0(x)$ and $r_1(x, y)$ for a two-step map defined in (3) take very simple forms namely

$$r_0(x) = \frac{1}{(2\pi)^2}\frac{\sin(sx)}{sx} \tag{76}$$

$$\begin{aligned} r_1(x, y) = {} & \frac{i(2\theta - 1)}{2s^3x^2y^2(x + y)}[sxy\{y\cos(sx)\sin(sy) - x\cos(sy)\sin(sx)\} \\ & + (x^2 - y^2)\sin(sx)\sin(sy)]. \end{aligned} \tag{77}$$

It can be seen that θ appears in r_1 but not in r_0. This means that the leading order of HO-GTE is independent of θ. Equation (77) also shows that $r_1(x, y)$ vanishes at $\theta = 1/2$ so that the leading order GTE is valid for long distances $O(1/z_a)$ if the positive and negative dispersion lengths of the fiber are the same. It can also be observed that

$$\lim_{s\to 0} r_0(x) = \frac{1}{(2\pi)^2}, \tag{78}$$

$$\lim_{s\to 0} r_1(x, y) = 0. \tag{79}$$

This shows that the higher-order GTE reduces to the ideal NLSE as the map strength approaches zero.

4.2 Lossy Case

For the lossy case, namely when $g(\zeta) \neq 1$, the kernel $r_0(x)$ depends on the relative position of the amplifier with respect to the dispersion map. The two step map given by $\Delta(\zeta)$ in (3) is considered and ζ_a is defined to represent the position of the amplifier within the dispersion map. So $|\zeta_a| < 1/2$ and $\zeta_a = 0$ means that the amplifier is placed at the mid point of the anomalous fiber segment. The function $g(\zeta)$ given by (11) can then be written as

$$g(\zeta) = \frac{2Ge^{2G}}{\sinh(2G)}e^{-4G(\zeta - n\zeta_a)} \tag{80}$$

for $\zeta_a + n \le \zeta < \zeta_a + n + 1$ where $G = \Gamma z_a/2$. The kernel $r_0(x)$ in the lossy case is computed in a similar method as in the lossless case. If $|\zeta_a| < \theta/2$, namely the amplifier is located in the anomalous fiber segment, the resulting expression for kernel is

$$\begin{aligned} r_0(x) = {} & \frac{1}{(2\pi)^2}\frac{Ge^{iC_0x}}{(sx + 2iG\theta)(sx - 2iG(1 - \theta))} \\ & \times \left[e^{G(4\zeta_a - 2\theta + 1)}sx\frac{\sin(sx - 2iG(1 - \theta))}{\sinh(2G)} \right. \\ & \left. + i\theta e^{i(4\zeta_a - 2\theta + 1)\frac{sx}{2\theta}}(sx - 2iG(1 - \theta))\right]. \end{aligned} \tag{81}$$

In (81), unlike the lossless case, the kernel $r_0(x)$ is complex and is explicitly dependent on the parameters θ, Γ, z_a and ζ_a in a nontrivial way. However, one still gets

$$\lim_{s\to 0} r_0(x) = \frac{1}{(2\pi)^2} \tag{82}$$

and moreover

$$\lim_{G\to 0} r_0(x) = \frac{1}{(2\pi)^2}\frac{\sin(sx)}{sx} \tag{83}$$

which means that as $z_a \to 0$ (76) is recovered. For the particular case $\theta = 1/2$, $\zeta_a = 0$ which corresponds to fiber segments of equal length with amplifiers placed at the middle of the anomalous fiber segment, the kernel modifies to

$$r_0(x) = \frac{1}{(2\pi)^2}\frac{G}{x^2s^2+G^2}\left[sx\frac{\sin(sx)}{\sinh(G)} + isx\left\{1-\frac{\cos(sx)}{\cosh(G)}\right\} + G\right]. \tag{84}$$

Also for $g(\zeta) \neq 1$, (75) gives

$$\lim_{s\to 0} r_1(x,y) = 0. \tag{85}$$

Thus, even in the lossy case, HO-GTE reduces to the case of ideal NLSE.

5 Conclusions

In this paper, the dynamics of vector optical solitons, propagating through optical fibers, with strong dispersion-management was studied. The birefringent fibers as well as multiple channels are considered. The technique that was used is the multiple-scale perturbation expansion. By using this technique the pulses in the Fourier domain was decomposed into a slowly evolving amplitude and a rapid phase that describes the chirp of the pulse. The fast phase is calculated explicitly that is driven by the large variations of the dispersion about the average. The amplitude evolution is described by the nonlocal evolution equations that is the HO-GTE.

These equations can be used to study the propagation of solitons with higher order accuracy, namely accuracy to $O(z_a^2)$. Also, the dynamics of quasilinear pulses [2, 3, 8], in optical fibers, can also be studied with greater accuracy. HO-GTE can also be used to study the four-wave mixing, timing and amplitude jitter and ghost pulses [5] for optical fibers, with better estimates and further accuracy than that was already obtained before. Better yet, HO-GTE can be used to study the detailed asymptotic properties governing the long-scale dynamics of optical pulses. It needs to be noted here that the derivation of the HO-GTE is valid for any arbitrary dispersion map $D(z)$ and with the general effects of damping and periodic amplification $g(z)$.

Although the HO-GTE is useful for studying the structure and properties, it is inconvenient for numerical computations because of the presence of the four-fold integrals that are given in the $O(z_a)$ terms of the HO-GTE. In the case of polarization preserving fibers, there was some numerical analysis done with special solutions of the HO-GTE and bi-solitons, tri-solitons and quartic-solitons was observed [4, 16, 17].

In future, one can extend this study to include the perturbation terms, for example, filters, higher order dispersion, Raman scattering, self-steepening just to name a few. Also, it is possible to take a look at the GTE and HO-GTE in the context of other laws of nonlinearity like parabolic law, saturable law and others.

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